2-Dimensional Finite Difference Time Domain

NGN 509 – Advanced Computational Methods

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Problem Statement

For the purpose of this project, we are interested in utilizing the 2-Dimensional (2D) FDTD method to solve Maxwell's equations numerically. There are two different derivations for the 2D FDTD method which are the transverse electric field set of equations (TE), and the transverse magnetics field (TM) equations. For the TM mode of equations, the magnetic fields are defined in the X and Y directions, and the electrical field in the Z direction. As for the TE mode of equations, the electrical fields are defined in the X and Y directions, and the magnetic field in the Z direction. The aim of this project is simulating a TM mode set of equations under different situations.

The TM mode equations are a set of 3 first order partial differential equations that will be solved using central difference in both the spatial and time domains to solve for the magnetic field in the x and y directions and the electrical field in the z direction. 3 simulations were performed utilizing the TM mode sets of equations. The first simulations involve simulating a point source of a set frequency and visualizing how the Electric field in the Z direction changes with respect to time. In this first simulation, the discontinuity that results from having a discrete solution space presents itself as a perfect reflective boundary that reflects back all fields that come in contact with it. Thus, this gives rise to the need for revising the derived equations. The revision is done with the addition of perfectly matched layers on each of the solution boundaries along the x and y directions. The added boundaries perform the task of meeting the desired absorptive boundary conditions by minimizing the magnitude of reflections that occur when the fields hit the boundaries. This is visualized in the second simulation by simulating the point source again with the updated set of equations. The final simulation involves simulating a horn antenna made of a conductive surface to control how the electromagnetic radiation spreads. In addition, a box with conductive properties was also added to simulate how the radiation bounces back from it. Such a box is known as a lossy propagation medium as some of the electromagnetic radiation is either scattered or absorbed instead of directly passing through it.

Introduction

Out of the four fundamental forces in nature, namely strong, weak, electromagnetic, and gravitational, the electromagnetic force has the greatest impact on technology. Among the three ways to forecast electromagnetic phenomena, namely experiment, analysis, and computation, computation is the most recent and rapidly developing method. Over the past decade, computational electromagnetics (CEM) has undergone remarkable progress, and it now provides extremely precise forecasts for several electromagnetically related problems, such as estimating the radar cross-section of targets and designing antennas and microwave devices with high accuracy [1].

Computational Electromagnetics is a field of electromagnetics specialized in solving Maxwell's equations computationally. Maxwell's equations are a set of fundamental equations that describe the interactions between electrical fields, magnetic field, and the mediums through which the combined electromagnetic field propagates through. Furthermore, Maxwell's equations are made up of 4 base equations that form the basis of classical electrodynamics. Computational Electromagnetics is a field of study involved with discretizing maxwell's equations and solving their different forms numerically. Multiply methods were created to perform this task such as the finite difference time domain method (FDTD), the finite difference frequency domain method (FDFD), the finite element method (FEM), and the method of moments (MoM).

Presently, there are two main classifications of commonly utilized CEM techniques. The first category is composed of differential equation (DE) methods, while the second category is made up of integral equation (IE) methods. Both DE and IE solution methods are rooted in the application of time-domain Maxwell's equations and the associated boundary conditions related to the problem that needs to be solved. In general, IE methods offer estimates for IEs using finite sums, while DE methods offer approximations for DEs in the form of finite differences. In general, numerical electromagnetics analysis has mainly been studied in the frequency domain, assuming time-harmonic behavior. This preference for the frequency domain approach was because of its capability of deriving analytical solutions to canonical problems, which served as a vital step in validating numerical results before implementing novel numerical techniques to obtain data for practical purposes. Additionally, the frequency-domain approach was favored as the hardware used in previous years for experimental measurements primarily supported this type of analysis [1].

Advanced computational resources have led to the development of more powerful time-domain CEM models. One popular time-domain approach is Differential Equations methods, which is favored for its simplicity and physical insight. Over the years, the Finite-Difference Time-Domain (FDTD) method became a widely used technique in CEM applications for antenna and microwave filter designs and analyzing the radar cross-section of 3D targets along with calculating the interaction of electromagnetic waves with bodies of different shape and different material. Additionally, the FDTD method has become a highly popular method of solving Maxwell's equations. It is based on simple formulations that eliminate the need for complicated asymptotic functions or Green's functions. Although the FDTD method solves problems in time, it can provide a vast frequency-domain response range using Fourier transform. It is capable of handling composite geometries that comprise various materials, including nonlinear, frequency-dependent, magnetic, dielectric, and anisotropic materials. With these features, the FDTD method has emerged as the most popular technique for many microwave devices and antenna applications in CEM and offers a more comprehensive understanding of underlying problems [2].

Maxwell Curl Equations

Maxwell's equations are a set of four equations that describe the behavior of electric and magnetic fields and how they relate to each other. The starting point for the construction of an FDTD algorithm is Maxwell's time-domain equations. The differential time-domain Maxwell's equations needed to specify the field behavior over time are:

$$
\begin{aligned}
\nabla \times \vec{H} &= \frac{\partial \vec{D}}{\partial t} + \vec{J}, \\
\nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} - \vec{M}, \\
\nabla \cdot \vec{D} &= \rho_e \\
\nabla \cdot \vec{B} &= \rho_m
\end{aligned}
$$

Where E is the electric field strength vector in volts per meter, D is the electric displacement vector in coulombs per square meter, H is the magnetic field strength vector in amperes per meter, B is the magnetic flux density vector in Webers per square meter, J is the electric current density vector in amperes per square meter, M is the magnetic current density vector in volts per square meter, P_e is the electric charge density in coulombs per cubic meter, and P_m is the magnetic charge density in webers per cubic meter [2].

FDTD Applications

Impedance sheets

In Radar Cross Section (RCS) scattering methods, thin sheets of resistive or dielectric material are often used. These sheets are also major components in waveguide and antenna designs and analysis. The analysis of such sheets can be simplified by using sheet impedances, which can conveniently and accurately model them in Finite-Difference Time-Domain (FDTD) calculations. Various approximate boundary conditions have been studied for thin sheets and layers. These conditions are applicable when the sheet's thickness is smaller than the free space wavelength, allowing them to be represented as an electric current sheet. If the sheet is mainly conductive, its impedance will be resistive, such as in the case of resistance cards. A lossless dielectric sheet will have purely reactive impedance, whereas generally speaking, the sheet impedance will be complex. Thin sheets are easily characterized by a discontinuity in the tangential magnetic field on either side, but no discontinuity in the tangential electric field, allowing the sheet current to be expressed in terms of an impedance multiplied by the electric field. These conditions imply that the perpendicular electric field's impact on the sheet can be disregarded, ensuring a single-valued behavior of the electric field. In summary, thin sheet analysis in RCS scattering methods and waveguide/antenna designs can be simplified by using sheet impedances. With the sheet impedance's accurate modelling, FDTD calculations can offer reliable solutions for these thin sheets [2].

The sheet impedance can be defined as the following:

$$Y_s = \sigma T + j\omega\varepsilon_0\left(\varepsilon_r - 1\right)T$$

With

$$Z_s = 1/Y_s$$

where Y_s is the sheet admittance, Z_s is the sheet impedance, sigma and er are the conductivity and relative permittivity of the sheet material, T is the sheet thickness, and ε_o is the free space permittivity.

Furthermore, let's explore the integration of this approximation into the FDTD technique. To utilize the surface impedance approximation, the impedance sheet thickness must be much smaller than the wavelength in free space. Typically, in FDTD computations, the FDTD cell size should be around 1/10 wavelength or less for reasonably accurate results, so this condition is automatically met. One can approximate an infinitesimally thin perfectly conducting plate as one FDTD cell thick to achieve good results. A similar approach can be used for impedance sheets by setting the thickness T as the FDTD cell thickness and adjusting the conductivity and/or relative permittivity in the FDTD calculations to achieve the desired sheet impedance. The FDTD cell thickness is only used to determine the conductivity and relative permittivity of the FDTD electric field location, which is important for approximating the desired sheet impedance [2].

Ground Penetrating Radar

Ground penetrating radar (GPR) is a geophysical tool using high frequency radio waves to probe inside a medium. In order to observe through opaque objects or beneath surfaces, it utilizes wide-band electromagnetic pulses. As engineering and environmental geophysics have developed over the recent years, GPR has taken on increasing value. It is now commonly used in a variety of studies, including bedrock structure detection, high building foundation evaluation, shallow soil or rock layer’s structure, underground water level detection, water pollution mapping, and other areas of study. Wave propagation in the subsurface material is more complicated than that in the air because of the physical characteristics of the geological material are varied and the electromagnetic attenuation in a subsurface material is greater than that in the air. For better comprehension and interpretation of GPR detection, it is effective to examine the radar wave propagation in such a material by utilizing FDTD technique. FDTD has been used to simulate wave propagation, scattering, and antenna radiation linked to GPR as a potent computational electromagnetic tool [3].

Theoretical Derivation

The Finite-Difference Time-Domain (FDTD) method is a numerical technique used to solve the Maxwell's equations for electromagnetic problems. The FDTD method is a powerful tool for solving electromagnetic problems in complex geometries and materials and is widely used in the design of electromagnetic devices and systems.

The first part of making an FDTD algorithm is Maxwell's time-domain equations, which are:

$$\begin{aligned} \nabla \times \vec{H} &= \frac{\partial \vec{D}}{\partial t} + \vec{J}, \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} - \vec{M}, \\ \nabla \cdot \vec{D} &= \rho_e \\ \nabla \cdot \vec{B} &= \rho_m \end{aligned}$$

where $\vec{E}$ is the electric field strength, $\vec{D}$ is the electric, $\vec{H}$ is the magnetic field, $\vec{B}$ is the magnetic flux density, $\vec{J}$ is the electric current, $\vec{M}$ is the magnetic current, ρ_e is the electric charge density, and ρ_m is the magnetic charge density. For our upcoming derivation, we assume that we are solving maxwell's equations for a source free region such that both the electric charge density and the magnetic charge density are zero.

The following constitutive relationship will supplement Maxwell's equations. Constitutive relations for linear, isotropic, and nondispersive materials are:

$$\begin{aligned} \vec{D} &= \varepsilon \vec{E}, \\ \vec{B} &= \mu \vec{H}, \end{aligned}$$

Where:

$$\begin{aligned} \varepsilon = \varepsilon_0 &\approx 8.854 \times 10^{-12} \text{ F/m}, \\ \mu = \mu_0 &= 4\pi \times 10^{-7} \text{ H/m} \end{aligned}$$

2-Dimensional FDTD updating equations for TM_z mode wave propagation

To derive FDTD equations, we only need to consider the curl equations (the first two of Maxwell's equations). The electric current density $\vec{J}$ is the sum of the conduction current density $\vec{J}_c = \sigma^e \vec{E}$ and the impressed current density $\vec{J}_i$ as $\vec{J} = \vec{J}_c + \vec{J}_i$. Similarly, for the magnetic current density, $\vec{M} = \vec{M}_c + \vec{M}_i$, where $\vec{M}_c = \sigma^m \vec{H}$. Here σ^e is the electric conductivity, and σ^m is the magnetic conductivity. However, for a source free region $\vec{J}_i$ and $\vec{M}_i$ are both zero. By making these substitutions, we arrive at these curl equations:

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} + \sigma^e \vec{E}$$
$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} - \sigma^m \vec{H}$$

This new formulation treats only the electromagnetic fields $\vec{E}$ and $\vec{H}$ and not the fluxes $\vec{D}$ and $\vec{B}$. Furthermore, we are only interested in the equations that specify transverse magnetic field propagation modes only. TM_z mode propagation means that the Electric field is propagating in the Z direction, which directly suggests that the magnetic fields exist in the X-Y plane due to their inherent orthogonality. Hence, we are only interested in $H_x, H_y, and\ E_z$. The derived Curl vector equations can be converted into six scalar equations for a three-dimensional space:

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon_x}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma_x^e E_x\right)$$
$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon_y}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma_y^e E_y\right)$$
$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon_z}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma_z^e E_z\right)$$
$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu_x}\left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} - \sigma_x^m H_x\right)$$
$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu_y}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} - \sigma_y^m H_y\right)$$
$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu_z}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} - \sigma_z^m H_z\right)$$

2-Dimensional FDTD updating equations with a Perfectly Matched Layer

A Perfectly Matched Layer (PML) is a technique commonly used in computational electromagnetics to simulate the effect of an infinite medium or to absorb the electromagnetic waves at the boundaries of a finite simulation domain. It has proven to be the most robust type of absorbing boundary condition used in computational electromagnetics.

The PML layer is a complex structure that is designed to gradually absorb the electromagnetic waves traveling through it, without reflecting them back into the simulation domain. The PML layer is anisotropic, meaning its properties are different in different directions, and is typically implemented as a set of additional FDTD grid points surrounding the computational domain. The layers add in loss in the form of both magnetic and electric conductivities whilst maintaining perfect impedance matching to allow the electromagnetic waves to pass through the PMLs whilst minimizing the magnitude of reflection back into the solution space. This effectively models the waves propagation into infinity instead of being confined in the limited solution space.

To incorporate the PMLs in the previously derived equations, the E_z is split up into parts that result from the x and y components of H. the electric and magnetic conductivities added have non-zero values only inside the PML layers. The electric conductivities in the PML layers σ_{pmy} & σ_{pmx} ,and the magnetic conductivities σ_{pmx} & σ_{pmy} are calculated such that the wave impedance stays the same as the electromagnetic radiation travels from the solution space to the PML layers such that:

$$\frac{\sigma_{pez}}{\varepsilon_0} = \frac{\sigma_{pmx}}{\mu_0}$$

$$\frac{\sigma_{pez}}{\varepsilon_0} = \frac{\sigma_{pmy}}{\mu_0}$$

Incorporating these added conductivities into the previously devised equations gives rise to:

$$\begin{aligned}
\varepsilon_0 \frac{\partial E_{zx}}{\partial t} + \sigma_{pex} E_{zx} &= \frac{\partial H_y}{\partial x}, \\
\varepsilon_0 \frac{\partial E_{zy}}{\partial t} + \sigma_{pey} E_{zy} &= -\frac{\partial H_x}{\partial y}, \\
\mu_0 \frac{\partial H_x}{\partial t} + \sigma_{pmy} H_x &= -\frac{\partial \left(E_{zx} + E_{zy}\right)}{\partial y}, \\
\mu_0 \frac{\partial H_y}{\partial t} + \sigma_{pmx} H_y &= \frac{\partial \left(E_{zx} + E_{zy}\right)}{\partial x}.
\end{aligned}$$

Numerical Formulation

In the previous chapter, we arrived at the following equations:

$$
\begin{aligned}
\frac{\partial E_z}{\partial t} &= \frac{1}{\varepsilon_z}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma_z^e E_z\right)\\
\frac{\partial H_x}{\partial t} &= \frac{1}{\mu_x}\left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} - \sigma_x^m H_x\right)\\
\frac{\partial H_y}{\partial t} &= \frac{1}{\mu_y}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} - \sigma_y^m H_y\right)
\end{aligned}
$$

In this set, the equations are dependent only on the terms H_x, H_y, and E_z, as all the magnetic field components are transverse to the reference dimension z; therefore, this set of equations constitutes the transverse magnetic to z case $-TM_z$.

2-Dimensional TMz FDTD updating equations without PMLs

To solve these equations using the FDTD method, the space is discretized into a grid of points, and the time is discretized into time steps. The values of Hx, Hy, and Ez at each point in the grid are updated at each time step based on the values of the neighbouring points, using finite-difference approximations of the partial derivatives. This discretized space is composed of cells named Yee cells.

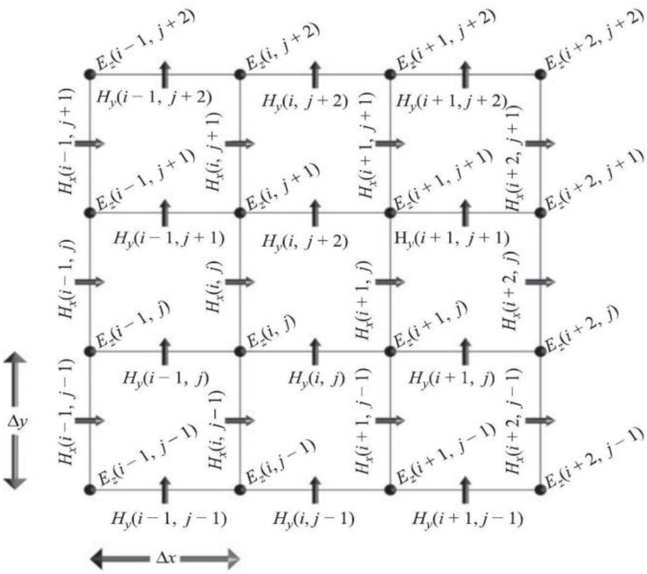

Fig.1. Yee cells for FDTD

The FDTD updating equations for the TM_Z case can be obtained by applying the central difference formula to the equations constituting the TM_Z case based on the field positions on the xy plane in the z direction as follows:

$$E_z^{n+1}(i,j) = A_{eze} \times E_z^n(i,j) + A_{ezhy} \times \left(H_y^{n+\frac{1}{2}}(i,j) - H_y^{n+\frac{1}{2}}(i-1,j) \right) + A_{ezhx} \times \left(H_x^{n+\frac{1}{2}}(i,j) - H_x^{n+\frac{1}{2}}(i,j-1) \right)$$

Where the update coefficients for Ez are defined as:

$$A_{eze}(i,j) = \frac{2\varepsilon_z(i,j) - \mathrm{dt}\sigma_z^e(i,j)}{2\varepsilon_z(i,j) + \mathrm{dt}\sigma_z^e(i,j)}$$

$$A_{ezhy}(i,j) = \frac{2\mathrm{dt}}{(2\varepsilon_z(i,j) + \mathrm{dt}\sigma_z^e(i,j))\mathrm{d}x}$$

$$A_{ezhx}(i,j) = -\frac{2\mathrm{dt}}{(2\varepsilon_z(i,j) + \mathrm{dt}\sigma_z^e(i,j))\mathrm{dy}}$$

And the update equation for Hx is simplified as:

$$H_x^{n+\frac{1}{2}}(i,j) = A_{hxh}(i,j) \times H_x^{n-\frac{1}{2}}(i,j) + A_{hxez}(i,j) \times (E_z^n(i,j+1) - E_z^n(i,j))$$

Where the update coefficients are:

$$A_{hxh}(i,j) = \frac{2\mu_x(i,j) - \mathrm{d}t\sigma_x^m(i,j)}{2\mu_x(i,j) + \mathrm{d}t\sigma_x^m(i,j)},$$

$$A_{hxez}(i,j) = -\frac{2\mathrm{d}t}{(2\mu_x(i,j) + \mathrm{d}t\sigma_x^m(i,j))\mathrm{dy}},$$

And lastly, the update equation for Hy is:

$$H_y^{n+\frac{1}{2}}(i,j) = A_{hyh}(i,j) \times H_y^{n-\frac{1}{2}}(i,j) + A_{hyez}(i,j) \times (E_z^n(i+1,j) - E_z^n(i,j))$$

Where the update coefficients are:

$$\begin{aligned} A_{hyh}(i,j) &= \frac{2\mu_y(i,j) - \mathrm{d}t\sigma_y^m(i,j)}{2\mu_y(i,j) + \mathrm{d}t\sigma_y^m(i,j)}, \\ A_{\mathrm{hyez}}(i,j) &= \frac{2\mathrm{d}t}{\big(2\mu_y(i,j) + \mathrm{d}t\sigma_y^m(i,j)\big)\mathrm{d}x} \\ A_{\mathrm{hym}}(i,j) &= -\frac{2\mathrm{d}t}{2\mu_y(i,j) + \mathrm{d}t\sigma_y^m(i,j)}. \end{aligned}$$

These equations summarize the update equations that are applied inside the solution space. where i and j denote the grid indices, n is the time step index, and dx and dy are the grid spacings in the x and y directions, respectively. However, to meet the absorptive boundary conditions, an additional set of equations is required to update the fields in the perfectly matched layers of the solutions space.

2-Dimensional TMz FDTD updating equations with PMLs

The PML updating equations can be obtained for the two-dimensional TM_z case by applying the central difference approximation to the derivatives in the modified Maxwell's equations.

$$E_{zx}^{n+1}(i,j) = A_{ezxe}(i,j) \times E_{zx}^{n}(i,j) + A_{ezxhy}(i,j) \times \left(H_y^{n+\frac{1}{2}}(i,j) - H_y^{n+\frac{1}{2}}(i-1,j) \right),$$

Where the update coefficients are:

$$\begin{aligned} A_{ezxe}(i,j) &= \frac{2\varepsilon_0 - \mathrm{d}t\sigma_{pex}(i,j)}{2\varepsilon_0 + \mathrm{d}t\sigma_{pex}(i,j)}, \\ A_{ezxhy}(i,j) &= \frac{2\mathrm{d}t}{\big(2\varepsilon_0 + \mathrm{d}t\sigma_{pex}(i,j)\big)\mathrm{d}x}. \end{aligned}$$

$$E_{zy}^{n+1}(i,j) = A_{ezye}(i,j) \times E_z^{n}(i,j) + A_{ezyhx}(i,j) \times \left(H_x^{n+\frac{1}{2}}(i,j) - H_x^{n+\frac{1}{2}}(i,j-1) \right),$$

Where the update coefficients are:

$$\begin{aligned} A_{ezye}(i,j) &= \frac{2\varepsilon_0(i,j) - \mathrm{d}t\sigma_{pey}(i,j)}{2\varepsilon_0(i,j) + \mathrm{d}t\sigma_{pey}(i,j)} \\ A_{ezyhx}(i,j) &= -\frac{2\mathrm{d}t}{\big(2\varepsilon_0 + \mathrm{d}t\sigma_{pey}(i,j)\big)\mathrm{d}y} \end{aligned}$$

$$H_x^{n+\frac{1}{2}}(i,j) = A_{hxh}(i,j) \times H_x^{n-\frac{1}{2}}(i,j) + A_{hxez}(i,j) \times \left(E_z^n(i,j+1) - E_z^n(i,j)\right)$$

Where the update coefficients are:

$$A_{hxh}(i,j) = \frac{2\mu_0 - \mathrm{d}t\sigma_{pmy}(i,j)}{2\mu_0 + \mathrm{d}t\sigma_{pmy}(i,j)},$$

$$A_{hxez}(i,j) = -\frac{2\mathrm{d}t}{\left(2\mu_0 + \mathrm{d}t\sigma_{pmy}(i,j)\right)\mathrm{d}y}.$$

$$H_y^{n+\frac{1}{2}}(i,j) = A_{hyh}(i,j) \times H_y^{n-\frac{1}{2}}(i,j) + A_{\mathrm{hyez}}\,(i,j) \times (E_z^n(i+1,j) - E_z^n(i,j)),$$

Where the update coefficients are:

$$A_{\mathrm{hyh}}\,(i,j) \quad = \frac{2\mu_0 - \mathrm{d}t\sigma_{pmx}(i,j)}{2\mu_0 + \mathrm{d}t\sigma_{pmx}(i,j)},$$

$$A_{\mathrm{hyez}}\,(i,j) \quad = \frac{2\mathrm{d}t}{\left(2\mu_0 + \mathrm{d}t\sigma_{pmx}(i,j)\right)\mathrm{d}x}.$$

Simulations

Now that we have derived the correct Update equations, we are able to simulate how the fields propagate through the solution space. The first step in performing the simulation using MATLAB is to define an appropriate solution space. This is performed by first picking our operating frequency which was set to 0.5GHz. after that, the solution grid was specified to have 10 wavelengths in each the x and y directions. The spatial steps dx and dy were set as one tenth of the wavelength to result in a 200x200 grid. Additional 20 PMLs were added to each side of the solution space to increase it into a 240x240 grid. The second step was to define the update coefficients, and initialize the fields to zero for t=0. The main program loop starts by updating the magnetic fields, then the electric fields, and lastly, both the magnetic fields and electric fields are updated in the PML layers. the Electric field is visualized at the end of each iteration to create an animation of how the Electric field changes with time. The time step was set as:

$$dt = \frac{1}{c * \sqrt{(dx^1 + dy^2)}}$$

The program flow is summarized below:

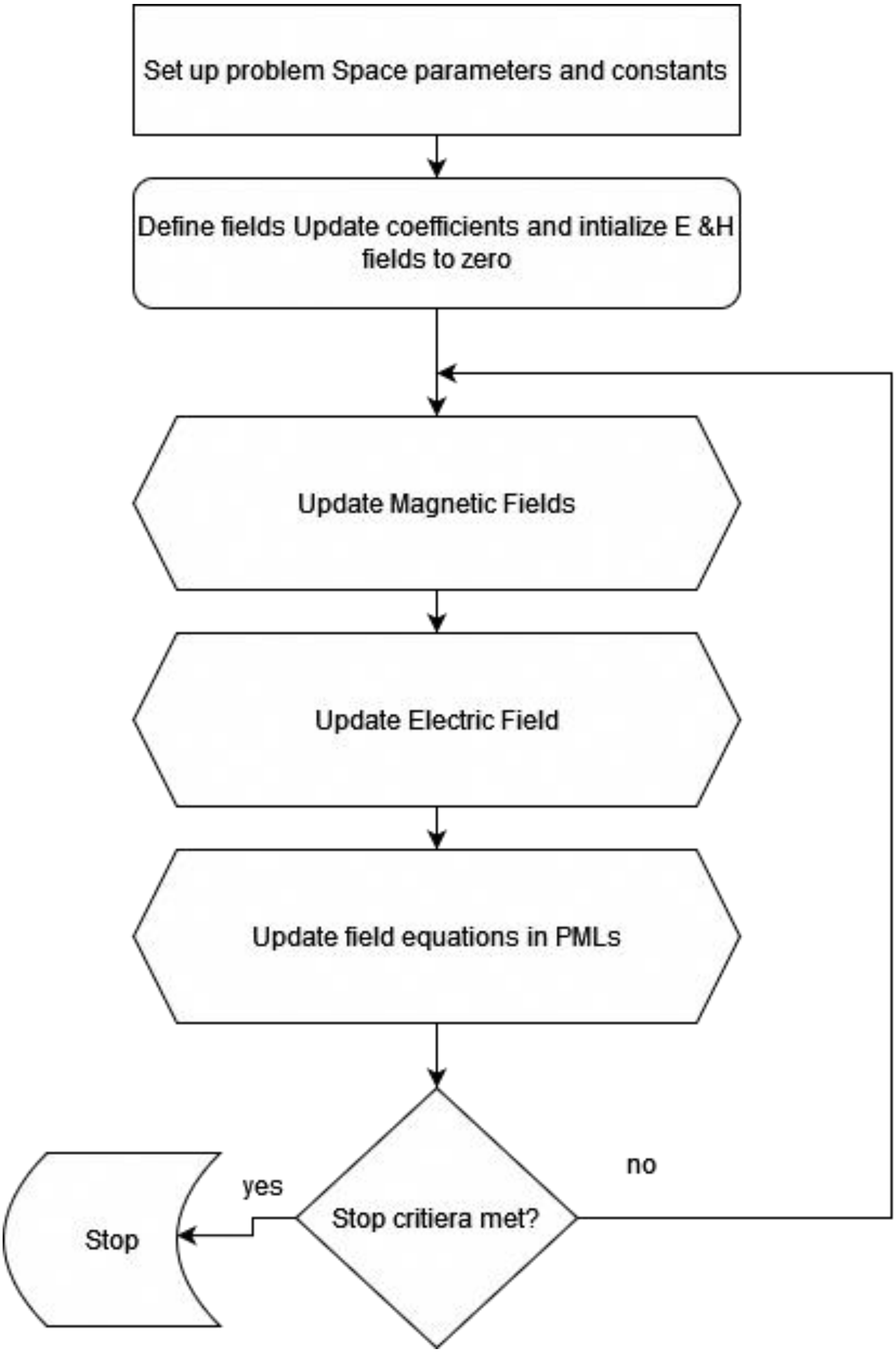

Fig.2. 2D FDTD program flow.

Results

The first simulation utilizes the update equations derived without setting up correct absorptive boundary conditions. The magnetic field in the x and y directions and electrical field in the z direction were initialized to zero. The time step size was defines as specified by the Courant–Friedrichs–Lewy convergence condition. The solution space was initialized to mimic vacuum conditions with the solution space being set as a 200x200 grid. A point source that radiates an electric field was simulated in the middle of the solution space with a frequency of 0.5 GHz. The grid was defined such that it has 10 wavelengths along each dimension with spatial step size of a tenth of the wavelength. Fig.3 shows how the Electric field is reflected from the solution space's boundaries as a result of the incorrect boundary conditions setup.

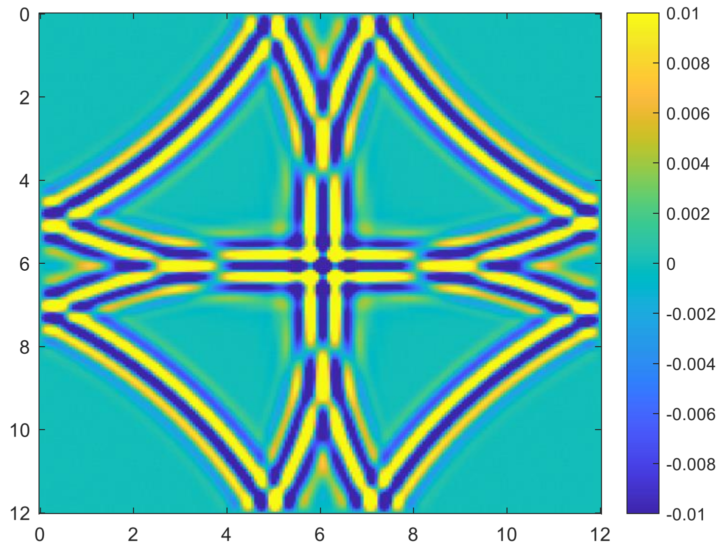

Fig.3. Electrical field reflection due the reflection solution boundaries.

The second simulation utilizes the revised update equations that include the correct absorptive boundary conditions. The same procedure was followed but with the addition of 20 PML layer in each boundary of the solution space to create a 240x240 grid. Fig,4 shows how the addition of the PML layers minimizes the reflections at the boundaries.

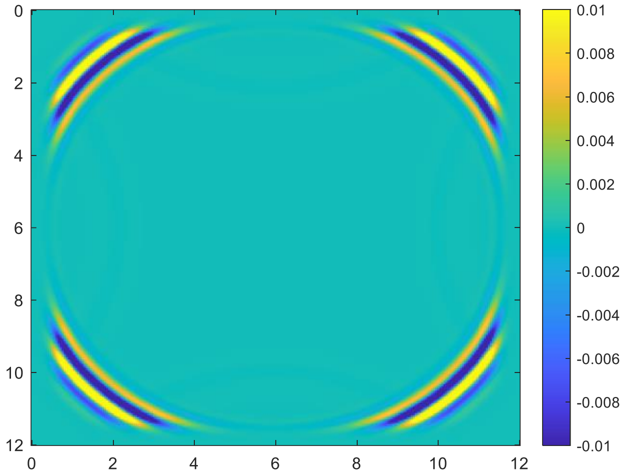

Fig.4. Electrical field absorption due to the addition of perfectly matched layers

For the third simulation, the point source was enclosed within a horn-like structure to direct the electromagnetic radiation. The horn like structure was first drawn and saved as a PNG file, and it was then imported into the solution space, The structure was modelled as a perfect electrical conductor which acts as a reflective surface which guides the electromagnetic radiation in a specific direction. A lossy box of low conductance and a different relative electric permittivity=3 was added to simulate the effects of waves scattering from a medium of low conductance in addition to travelling through a medium of a different electric permittivity. Fig.5 shows how the electric field is confined to exit from the opening in the horn only as seen by the lower field intensities outside the horn. Further, it is clear that the speed of the electromagnetic field is slower inside the box due to the effects of the increase electrical permittivity. Lastly, it is clear that some of the electromagnetic radiation is scattered back by the box due to its low conductivity, and the difference in relative permittivity between vacuum and the lossy box.

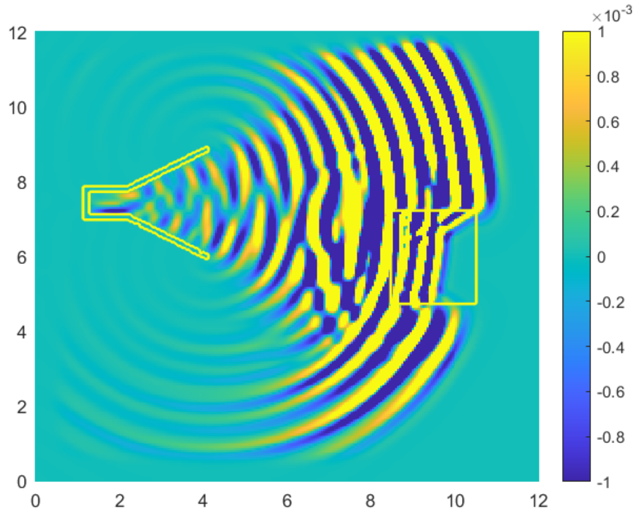

Fig.5. Horn structure and lossy box simulation

Conclusion & Discussions

This project aims to use computational techniques to simulate a problem in electrical engineering. The 2D finite difference time domain method was utilized to solve a modified version of maxwell's equations to solve a transverse magnetic field 2D problem. the theoretical derivation for the TM mode set of partial differential equations was performed, and the computational update equations were derived using central difference in both the spatial and time domains. The update equations were then revised to include absorptive boundary conditions in the form of perfectly matched layers. 3 simulations were performed to verify that the formulated update equations work. The first simulation was of a point source that radiates an electrical field in the middle of the solution space without the addition of the PMLs. This resulted in the electromagnetic fields reflecting endlessly inside the solution space. The second simulation is of the same point source but with the addition of 20 PMLs in each of the solution space's boundaries. The results indicate that most of the electromagnetic radiation is absorbed and minimal reflection is observed back into the solution space. Lastly, a simulation in which a horn like structure modelled as a perfect electric conductor is created in addition to a lossy box were added to the solution space to observe how the electromagnetic radiation interacts with it.

One of the main limitations faced is that grid size had be relatively large compared to the wavelength of the excited point source. Hence, the resolution of the simulation is low but acceptable. Increasing the solution space's dimension would result in a great increase in the time required to finish the simulation. However, this might have been solved if interpolation techniques were used to interpolate additional Yee-cells to increase the resolution. Another improvement that could have been made is to model the electromagnetic radiation source as a current/voltage that is applied to a conductive surface. This would have helped in creating a more realistic simulation. Lastly, exploring the 3D FDTD method would have enabled the simulation of real-world structures and how they interact with electromagnetic radiation.

References

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